Lowest Common Multiple, Highest Common Factor and Prime Factorisation Problems - **Answers**

This is a non-calculator exercise

1. Lane strikes her drum every 9 seconds and Rory plucks her guitar every 15 seconds. They strike and pluck at the same time at the beginning of the song. After how many seconds will they next strike and pluck together?
   45 seconds

2. David has a length of ribbon which is 180cm long. Kylie's length of ribbon is 225cm long. They want to cut the longest lengths of ribbon they can from their lengths without wasting any, and in such a way that every piece of ribbon they cut is equal in length. How long will the lengths be?
   45cm

3. Express 960 as the product of prime factors. Use your result to show that 15 is the largest odd factor of 960.
   $2^6 \times 3 \times 5$
   3 and 5 are the only odd factors so 3×5 is the largest odd factor.

4. The children at Freeferal Primary School are going on a school trip to Blackpool and every pupil who wants to attend must pay to do so. Mr N Rooly collects £255 from his pupils. Mrs K Ottick collects £136 from her pupils. If the cost is a whole number of pounds, what is the highest it could be?
   HCF = 17. £17 is the highest cost.

5. The highest common factor of two numbers is 32. The lowest common multiple of the two numbers is 640. What could the two numbers be?
   160 and 128
   or 32 and 640

6. Cain washes his bedding every 4 weeks. Chas washes her bedding every 5 days. If they both wash their bedding on April 1st, what is the next date they will both wash their bedding?
   LCM (28, 5) = 140
   They both wash their bedding on 18th August.

7. By expressing it as the product of prime factors, show that 5625 is a square number.
   $5625 = 5 \times 5 \times 5 \times 5 \times 3 \times 3 = (5 \times 5 \times 3)^2$
   So $5625 = 75^2$

8. Use prime factorisation to show that 27 000 is a cube number and find its cube root.
   $27\,000 = 2^3 \times 3^3 \times 5^3 = (2 \times 3 \times 5)^3 = 30^3$

9. Sue-Lynn buys some packets of biscuits; each contains the same number of biscuits. She opens the packets up and counts the biscuits. She has 209 biscuits. How many packets could she have bought?
   209 = 11 × 19
   She could have bought 11 or 19 packets.

10. What is the smallest number that is divisible by 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 and 10?
    1;2;3;4 = (2 × 2);5;6 = (2 × 3);7;8 = (2 × 2 × 2);9 = (3 × 3);10 = (2 × 5)
    LCM(1,2,3,4,5,6,7,8,9,10) = 2520

11. For any number, m, m! is the product of all positive whole numbers from m itself down to 1.
    For example:
    $4!=4\times3\times2\times1$
    If $n!=2^8\times3^4\times5^2\times7$, find the value of n.~
    $2^8 \times 3^4 \times 5^2 \times 7 = 1 \times 2 \times 3 \times (2 \times 2) \times 5 \times (2 \times 3) \times 7 \times (2 \times 2 \times 2) \times (3 \times 3) \times (2 \times 5)$
    therefore n = 10

Lowest Common Multiple, Highest Common Factor and Prime Factorisation Problems

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For example:
$4!=4\times3\times2\times1$
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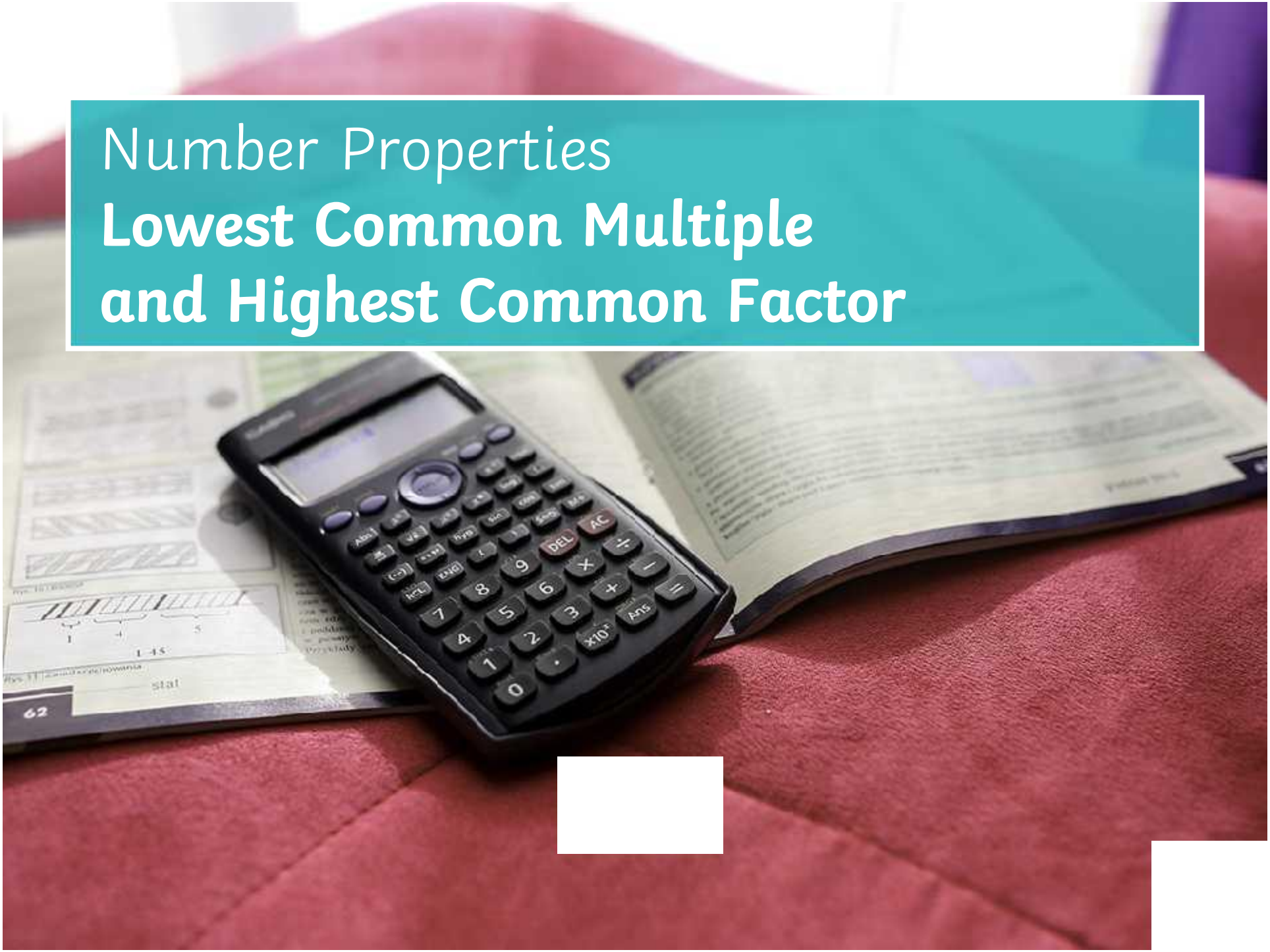


Number Properties
Lowest Common Multiple and Highest Common Factor

Learning Objective

- To understand and be able to solve problems involving highest common factors, lowest common multiples and prime factorisation.

Success Criteria

- To find the HCF and LCM of a pair of numbers using lists.
- To express numbers as products of their prime factors.
- To apply prime factorisation to finding the HCF and LCM, as well as solving problems involving properties of numbers.

Starter: Using Lists to Find the Lowest Common Multiple (LCM)

The multiples of 15 are:
15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, ...
The multiples of 12 are:
12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, ...

60, 120 and 180 are common multiples of 15 and 12.
(Common means that they are shared by both 15 and 12.)
The lowest common multiple of 12 and 15 is 60.
We say that LCM (12, 15) = 60

Why would you never be asked to find the highest common multiple of two numbers?

Now find the LCM of 8 and 6.

The multiples of 8 are 8, 16, 24, 32, 40, 48, ...
The multiples of 6 are 6, 12, 18, 24, 30, 36, ...
The LCM of 8 and 6 is 24.

Starter: Using Lists to Find the Highest Common Factor (HCF)

The factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

The common factors of 30 and 24 are 1, 2, 3, and 6.
The highest common factor of 30 and 24 is 6.
HCF (30, 24) = 6

Why would you not usually be asked to find the lowest common factor of two numbers?

Now find the HCF of 14 and 21.

The factors of 14 are 1, 2, 7 and 14.
The factors of 21 are 1, 3, 7 and 21.
The HCF of 14 and 21 is 7.

Using a Factor Tree for Prime Factorisation

If you are asked to find 30 as the **product of prime factors**, you are aiming to find the prime numbers which multiply together to give 30.

A factor tree is a good way to prime factorise a number.

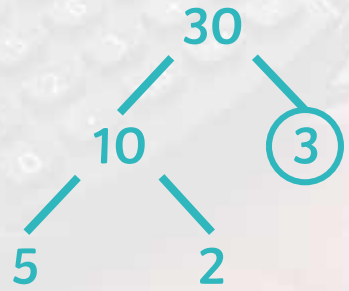
Begin with the number 30, draw two lines coming out of it and think of two numbers which multiply to give 30; do not use the number 1.

Using a Factor Tree for Prime Factorisation

Now, circle any numbers that are prime. In this example, 3 is the only prime number so far.

Where a number is not prime, you will need to find 2 numbers that multiply to give that number; neither of them should be 1.

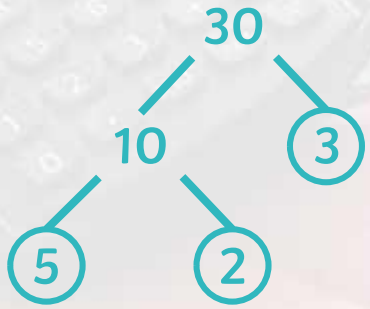
10 is not prime. 5 multiplied by 2 is 10, so we write those numbers at the end of the lines coming out of 10.



Using a Factor Tree for Prime Factorisation

Now, again, circle any numbers that are prime, in this case, the 5 and the 2.

When all of the numbers at the ends of the lines are circled, the factor tree is complete. The circled numbers are the prime factors.
30 = 2 x 3 x 5, written as the product of prime factors.



Prime Factorisation

Working independently, use a factor tree to write 480 as the product of prime factors.

Neither 48 nor 10 are prime, so we draw lines coming out of each. 8 multiplied by 6 is 48 and 5 multiplied by 2 is 10, so we put those numbers at the end of our lines.

You may have split 480 differently but you should end up with the same final answer. We can begin by splitting it into 48 and 10.

At the end of our lines, we now have 6, 8, 2 and 5. 2 and 5 are prime, so we circle them. We must think of pairs of numbers which multiply to give 6 and 8.

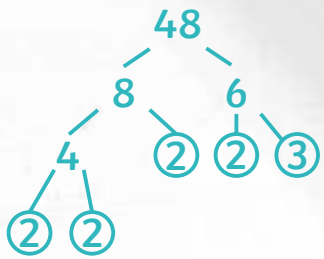
We need to circle the 2s and the 3 and find a way of expressing 4 as a product; that would be 2 multiplied by 2. Those 2s should also be circled as they too, are primes.

So 480 = 2 x 2 x 2 x 2 x 2 x 3 x 5, written as the product of primes.

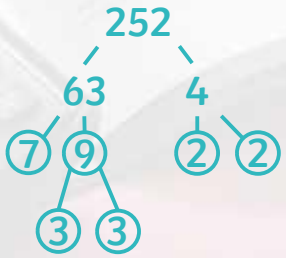
Written in index notation: $480 = 2^5 \times 3 \times 5$

Using Prime Factorisation to Find HCF and LCM

Prime factorisation can be used to find the lowest common multiple or the highest common factor of two or more numbers. We will use prime factorisation to find the highest common factor and the lowest common multiple of 48 and 252.



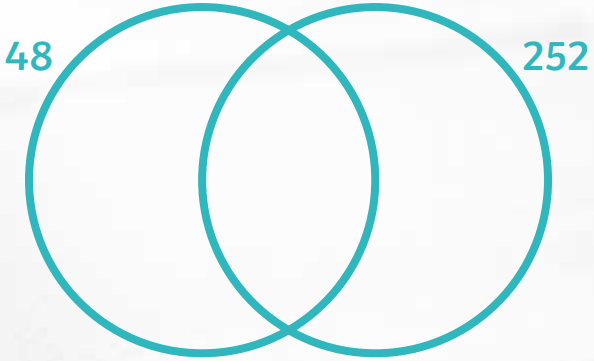
Creating a prime factor tree for 48, we see that $48 = 2^4 \times 3$



Creating a prime factor tree for 252, we see that $252 = 2^2 \times 3^2 \times 7$ as the product of primes.

Using a Venn Diagram to Find HCF and LCM

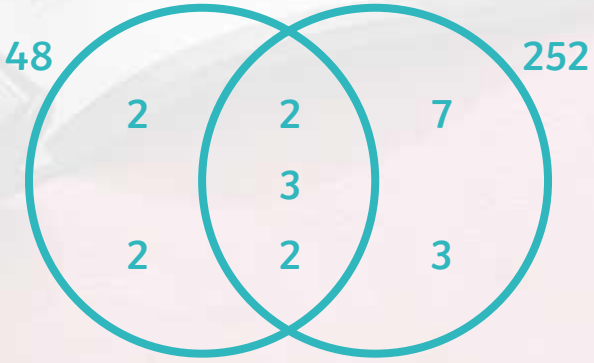
Draw 2 overlapping circles and label them 48 and 252.



Enter the prime factors into the relevant places within the circles.

$48 = 2^4 \times 3$

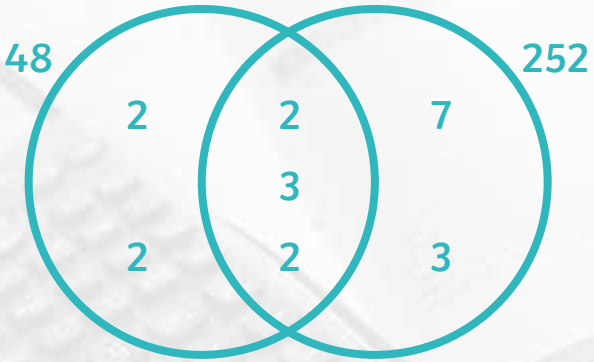
$252 = 2^2 \times 3^2 \times 7$



Notice that two 2s and a 3 are in the overlap because these are the factors that are common to both. The other factors are placed just within one circle, not the overlap.

Using a Venn Diagram to Find HCF and LCM

From this diagram, we can find the highest common factor and the lowest common multiple of 252 and 48.



To find the highest common factor, we take all of the numbers in the overlap and multiply them together.
HCF (252, 48) = 2 x 2 x 3 = 12

For the lowest common multiple, we take all of the numbers inside the diagram and multiply them together.
LCM (252, 48) = 2 x 2 x 2 x 2 x 3 x 3 x 7 = 1008

Using Prime Factorisation to Solve Problems

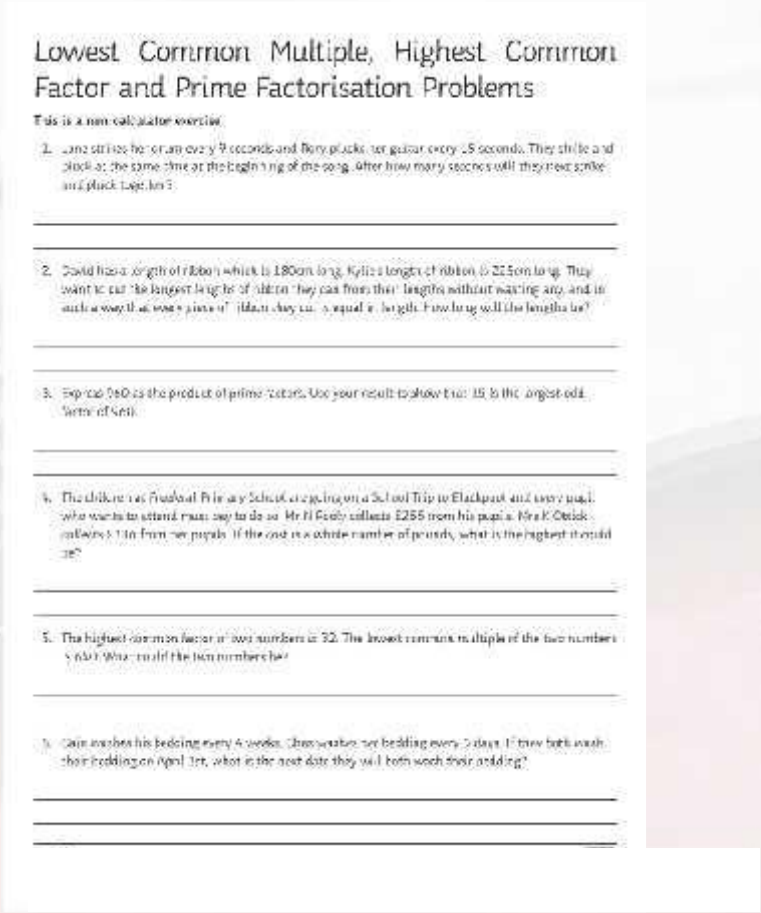
Use prime factorisation to show that 3136 is a square number.

This can be written as $2 \times 2 \times 2 \times 7 \times 2 \times 2 \times 2 \times 7$

This can be written as $(2 \times 2 \times 2 \times 7)^2$ and $2 \times 2 \times 2 \times 7 = 56$ so $3136 = 56^2$

Activity Sheet

Now work through the exercises on the Activity Sheet.



Lowest Common Multiple, Highest Common Factor and Prime Factorisation Problems

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3. Express 360 as the product of prime factors. Use your result to show that 15 is the largest odd factor of 360.

4. The children at Foxhall Primary School are going on a School Trip to Blackpool and every pupil who wants to attend must pay to do so. Mr H Peaty collects £255 from his pupils. Mrs K Obtick collects £136 from her pupils. If the cost is a whole number of pounds, what is the highest it could be?

5. The highest common factor of two numbers is 32. The lowest common multiple of the two numbers is 640. What could the two numbers be?

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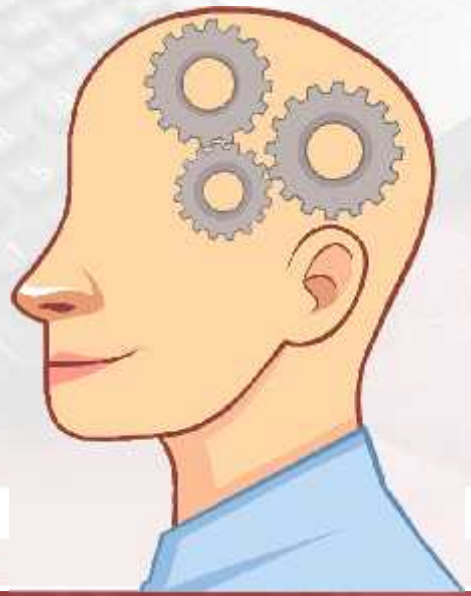
Plenary: Finding a Factor with a Given Number Property

Use prime factorisation to find the highest factor of 2835 which is not a multiple of 3.

$2835 = 3^4 \times 5 \times 7$

$5 \times 7 = 35$ is the highest factor of 2835 which is not a multiple of 3.

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Lowest Common Multiple and Highest Common Factor
Teaching Ideas

Learning Objective:	To understand and be able to solve problems involving highest common factors, lowest common multiples and prime factorisation.
Success Criteria:	• To find the HCF and LCM of a pair of numbers using lists. • To express numbers as products of their prime factors. • To apply prime factorisation to finding the HCF and LCM, as well as solving problems involving properties of numbers.
Context:	This lesson can be used as Lesson 10 as part of the Place Value Scheme of Work. It assumes no knowledge about HCFs, LCMs or prime factorisation and carefully explains and demonstrates these concepts. This lesson would also work well as a brief review of HCF and LCM and an introduction to prime factorisation as a means of determining properties of numbers.

Starter

Derive the definitions of LCM and HCF with students by asking them to consider the words: multiples, factors, common, lowest and highest. Demonstrate how to find the LCM and HCF using lists then have the students find the LCM and HCF of pairs of numbers independently before going over the answer. You may wish to take this opportunity to ask students to think of applications of lowest common factors and highest common mulitples. They may come up with factorising in algebra, simplifying fractions and ratios, solving problems involving recurring events with given frequencies, e.g. two trains departing regularly but on different schedules, adding fractions or saying "fizz buzz" in the game fizz buzz.

Main Activities

Prime Factorisation

Demonstrate the factor tree method for the prime factorisation of the number 30. Allow students to use the method to express 480 as the product of its primes, then go over the method and answer. Did any of the students think to write their answer in index notation?

Using a Venn Diagram to find HCF and LCM

Use the slides to demonstrate, step by step, how to use a Venn diagram and prime factors to find the HCF and LCM of a pair of numbers. Can the students come up with an example where one section of the diagram would be completely empty?

Using Prime Factorisation to Solve Problems

Give students an example of a problem-solving activity using prime factorisation then have them work individually through the **Lowest Common Multiple and Highest Common Factor Activity Sheet**, which gives them experience of a variety of questions, building carefully on skills acquired. An **Answer Sheet** is also provided.

Plenary

Have students attempt to solve the problem about the highest factor which is not a multiple of 3, using prime factorisation. Bring the class together to discuss the solution.